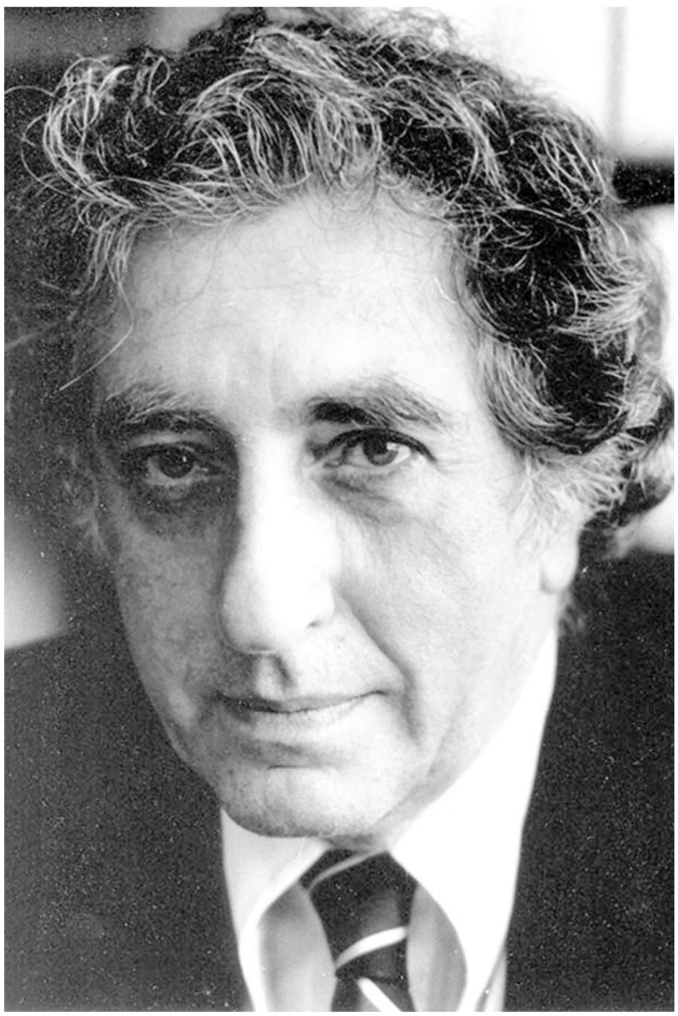
Cyber profiles

Dr George Adomian – distinguished scientist and mathematician

R.C. Rach

Asked about his decomposition method, Adomian says it is simplicity itself, requires no abstruse mathematics but only freeing one's mind from the ingrained methods and prejudices one learns in universities, and the idea that one must follow the same road as those who have gone before. In one of his books, he tells students "Do not follow in the footsteps of the masters"!



It is with great regret that we have to record the death of Dr Adomian on 18 June 1996. It has been decided that this profile should continue to be published since it was compiled with the co-operation of his friends and colleagues. It will be but one of the very many posthumous tributes to his life's work and immense contributions to cybernetics.

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Some have found it hard to believe that a sum of non-linear functions can converge exactly to a given non-linearity. Adomian says it is somewhat like a Fourier series but instead of sines and cosines, special polynomials are generated to represent specific non-linearities. Actually this series of Adomian polynomials is a generalization of Taylor series which subsumes the regular Taylor series. He departs from the beginning from the time-honoured Picard form and substitutes an operator equation, which is far more general and useful. The method is certainly simpler than finite differences or finite elements, for example. With decomposition, one can solve systems of non-linear operator equations in one to four dimensions as easily as a linear ordinary differential equation. In fact, courses on ordinary and partial differential equations and non-linear systems of equations can now be combined into a single course. It has been applied to algebraic equations, integral equations, delay-differential equations, and integro-differential equations as well as a host of applications.

He has long been concerned about the total reliance in physics and mathematics on linearity and determinism in a world clearly non-linear and stochastic in general. Non-linearity and stochasticity are present in varying degree in all physics but are largely ignored or inadequately treated because of mathematical limitations. Randomness had been treated by hierarchy methods involving unjustified closure approximations, or equivalently, perturbation theory, or finally by simple neglect, i.e. the assumption that equations are always deterministic. In 1963, in his doctoral dissertation, and in prior and subsequent work, he treated these problems. The need for such treatment was evidenced by interest in publication of his dissertation as a special issue or monograph. In the 1960s and early 1970s Adomian introduced concepts of "stochastic operators" and of "stochastic differential equations" wherein parameters, inputs, or conditions could be stochastic. His work expanded these subjects for a number of years. Then, in 1976, Adomian published a major result in non-linear systems with a solution of non-linear differential equations with polynomial non-linearities involving neither linearization or perturbation. This was published in a journal and also presented at the Norbert Wiener Symposium and in *Modern Trends in Cybernetics and Systems* edited by J. Rose.

After 1976, the class of non-linearities was rapidly expanded with the further development of the Adomian Polynomials. The Decomposition Method, as it is now called, is solving wide classes of ordinary and partial differential equations without linearization or perturbation, or the intensive computation of the discretized methods. It has now been applied in mathematics, physics, astronomy, geology, biology, medicine, pharmacology, hydrology, ecology, and economics. The First Congress on Mathematical Models and Methods Applied to Biology and Medicine, to be held in Alicante, Spain, on 30 June to 3 July 1997, will be dedicated to the memory of Dr Adomian. He has published well over 300 papers, one translation and six books, with a seventh to be published. His achievements also include fundamental contributions to modern military defence and weapons systems, including radar, aircraft control and high-energy lasers.

Adomian is a Fellow of the AAAS, member of SIAM, AMS, APS, Sigma Xi, Tau Beta Pi, Eta Kappa Nu, and Sigma Pi Sigma. He was a National Academy of Sciences – USSR Academy of Sciences Scholar in 1988. He is listed in *Who's Who in the World, Who's Who in America, Who's Who in Science and Technology,* etc.

He was presented the Richard E. Bellman Prize for outstanding contributions to non-linear stochastic analysis in 1989.

Academic vita

Position:	Chief Scientist, General Analytics Corporation, 155 Clyde Rd, Athens, Georgia 30605.
Degrees:	PhD, Theoretical Physics, University of California (Los Angeles), 1963; BS, MS, Electrical Engineering, University of Michigan; further study in mathematics, University of Michigan and CALTEC.
Research:	Non-linear stochastic dynamical systems, differential and partial differential equations, applied mathematics and theoretical physics.
Experience:	Distinguished Professor of Mathematics (David Crenshaw Barrow chair); Director, Center for Applied Mathematics; Research Professor of Engineering, Professor of Mathematics; Senior Scientist (aerospace)
Author:	*Linear Stochastic Operators*, PhD dissertation, UCLA, 1963. *Applied Stochastic Processes*, Academic Press, 1980. *Stochastic Systems*, Academic Press, 1983. *Stochastic Systems* (Russian translation), Mir Publishers, 1987. *Non-linear Stochastic Operator Equations*, Academic Press, 1986. *Non-linear Stochastic Systems Theory and Applications to Physics*, Kluwer, 1989. *Partial Differential Equations: New Methods for Their Treatment and Solution*, D. Reidel, 1985 (R.E. Bellman, co-author). *Solving Frontier Problems of Physics – The Decomposition Method*, Kluwer, 1994. *Non-linear Theory and Applications in Mathematical Physics: The Decomposition Method*, to appear, 1997. Over 300 journal papers. Numerous conference papers and classified papers. A list of "selected publications" may be obtained from the editor of *Kybernetes*.
Awards:	1989 Richard Bellman Prize for outstanding contributions to non-linear stochastic analysis; AAAS Fellow; National Academy of Sciences Scholar 1988; Howard Hughes Doctoral Fellow; Sigma Xi, Tau Beta Pi, Eta Kappa Nu, Sigma Pi Sigma.
Listings:	*Who's Who in America, Who's Who in the World, Who's Who in Science and Technology*, etc.
Other activities:	Defence contracts, defence committees and panels, intensive short courses for government agencies and industry, Senior Associate, National Science Center Foundation.

On George Adomian's outstanding contributions

Professor Yves Cherrault (Professor at the Université Pierre et Marie Curie, and Director of MEDIMAT, Paris, France) writes:

I have known Professor George Adomian for a long time. I first met him in a scientific meeting in 1985 after discovering his new decomposition method in a paper published by *Kybernetes*. I was very surprised by this original and creative technique for solving non-linear functional equations because it worked without discretization and without linearization of the non-linear term! At this time the convergence of the method seemed very mysterious and it became a challenge for me. I first proved the convergence by using the fixed point theorem after giving a new expression of the method. Then one of my most gifted researchers, Karim Abbaoui, gave new and alternative formulae for calculating the Adomian polynomials and for proving convergence in more general situations.

Some years ago George Adomian came to Paris and gave some lectures in my laboratory. We have also had very fruitful discussions about decomposition methods and their applications to the resolution of partial differential equations.

George Adomian became for me a valued and dear friend but he was also a first class scientist and from my point of view one of the best applied mathematicians of his generation, able to deal effectively with a wide range of frontier applications. Many people have been working (for a long time) on methods for solving non-linear functional equations but George Adomian was the first to propose an *original* and *powerful* method for obtaining a solution without discretization of space or time and without linearization of the non-linear operator. Many unsolved problems can be treated with success by this new method.

In summary, Professor George Adomian was an outstanding scientist of international repute.

Paris, 6 February 1996

Richard Bellman

Richard Bellman's preface to Dr Adomian's 1986 book is not only an interesting assessment of work but also in itself an acknowledgement of what he describes as a "scientific breakthrough, the importance of it cannot be emphasized". He wrote:

> A previous volume, *Stochastic Systems* (1983), was the first systematic book bringing non-linear and stochastic equations within the reach of engineers and physicists concerned with the difficulties of real systems and frontier problems in which conventional techniques such as linearization and perturbation are not sufficient or realistic.
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> This volume increases very substantially the scope of the work to cover very wide classes of equations. It is a simple and powerful method applicable to the solution of a multitude of problems in physics. engineering, operations research, systems analysis, economics, biology, medicine, and other fields.
>
> Forthcoming volumes will deal with applications to physics and engineering. Previously, either such problems were not realistically formulated or the approximations made and the methods used made the results either unrealistic or misleading. This work will necessitate the rewriting of the mathematics and physics books since mathematics deals largely with linear operation theory and physics with perturbation. In my view, this overall work will have a profound impact on mathematics and on the modelling of problems of physics, engineering, economics, biology, and medicine and may well be viewed in retrospect as one of the significant contributions of this century. This work is a scientific breakthrough. The importance of it cannot be over-emphasized.

H.J. Efinger

Finally, a more recent review of his book, *Solving Frontier Problems of Physics: The Decomposition Method*, published by Kluwer Academic Publishers in 1994, will give readers of *Kybernetes* an insight into his new mathematical method which bears his name – the Adomian Decomposition Method. The reviewer, H.J. Efinger, writes:

The basic idea underlying Adomian's method is of ingenious simplicity with far reaching applications. Workers in the field of, say, integro-differential equations are very familiar with power series expansions and the formalism of Green's functions or the general use of integral-kernels, all of which is called standard mathematical analysis. But how do we handle complicated integrations via Green's functions? It is true that certain methods (Mercer's theorem comes to mind, etc.) are applicable to a wide class of frontier problems in classical and modern physics.

To this end let us consider the following equation in one independent variable $t \in IR$: $F_u = g(t)$; here F is a non-linear ordinary differential operator with linear and non-linear terms. For example, let F be of second order; for some initial-value problem we have $u = u(0) + t_u'(0) + \tilde{L}^{-1}[g - Nu]$, provided $\tilde{L}^{-1}$ exists, where $\tilde{L}$ is the linear and N is the non-linear part of F. It is an elegant way of hiding our ignorance about the actual integration process by saying that the linear operation $\tilde{L}$ is invertible, i.e. some Green's function exists. To highlight Adomian's gambit, we simply put $\tilde{L} = (L + R)$; here L is the highest order derivative (in the present example d^2/dt^2), and R is the linear remainder (incidentally, a linear stochastic operator can be included); then in general L^{-1} is just an n-fold definite integration operator from zero to t (for a boundary-value problem L^{-1} would be an indefinite integral). Now decomposition means that the solution is assumed to be $u = \Sigma_{n=0}^{\infty} u_n$, with $u_0 = u(0) + tu'(0) + L^{-1}g$.

The next step in the decomposition procedure is the evaluation of Nu, being some non-linear function $f(u)$: Let f be analytic, such that $f = \Sigma_{n=0}^{\infty} A_n$, where the sum is taken over polynomials A_n depending only on the u_0 to u_n components; then Adomian shows that $\Sigma A_n = \Sigma[(u - u_0)^n/n!]f^{(n)}(u_0)$. This is a generalized Taylor series about the starting function u_0 (as opposed to a series about a given point): the exact solution can be written as $u = \lim_{n\to\infty} \Sigma_{i=0}^{n-1} u_i$, with $u_i = -L^{-1}(Ru_{i-1} + A_{i-1})$, $i \geq 1$, the A_i (called Adomian-polynomials) are consistently arranged, and ensure rapid convergence of the partial sum $\Sigma_{i=0}^{n-1} u_i$ which serves as a practical solution.

Adomian's findings (generalized to several variables) are almost uncanny. And yet his approach is but another lesson as to the richness of basic mathematical reasoning.

I am certain that this book is a very important one since Adomian's method is now well developed and solves ordinary as well as partial differential equations for extremely general (linear, non-linear, deterministic, stochastic, or even strongly coupled) initial/boundary conditions. It will certainly affect graduate courses and frontier research in science and engineering. Richard Bellman was very familiar with Adomian's work and foresaw its impact – now finally accelerating. What I like especially is that solving ordinary or partial differential equations (including non-linear dynamical systems such as Navier-Stokes equations, many-body problems and non-linear coupled wave-type equations) becomes just one method instead of a collection of approximation techniques.

In chapter 1 Adomian presents his philosophy on modelling physical phenomena: in his words, "Modelling is necessarily a compromise between physical realism and our ability to solve the resulting equations", whereby any incentive for more accuracy should avoid altogether linearization/perturbation-techniques, etc. Instead, the objective must be, so he adds, "to model a problem with its inherent non-linearities and random fluctuations or uncertain data".

In the opening chapters (up to 5) the decomposition method in one and several variables is reviewed and extended beyond previous presentations (by Adomian's earlier books). In these and subsequent chapters important references are given and further reading is suggested. I find it extremely useful that specific working examples with error estimates as to the convergence of the decomposition method are provided and contrasted with alternative or even less effective methods.

In the following chapters (up to 9), general applications of the decomposition method are given in more detail, encompassing solutions for Neuman Integral and boundary conditions at infinity. In chapter 10 the applicability of the decomposition method is discussed to linear and non-linear integral-equations in terms of instructive examples.

In chapter 11 the general scheme for non-linear oscillations in physical systems (Duffing, Van der Pol, etc.) is treated by the decomposition method, and in chapter 12 the solution of the Duffing equation is considered with special care on inhomogeneous input-functions.

In chapter 13 the modelling of physical boundary-value problems is presented with closed contours or surfaces: The objective is to solve "two-limit" boundary-value problems which is the analogue of solving two-point boundary-value problems for ordinary differential equations with Dirichlet conditions. It is made clear that in two dimensions one deals with a two-contour second-order partial differential equation, whereas in three dimensions the corresponding partial differential equation is solved between two surfaces.

In the final chapter 14 Adomian treats selective topics of fundamental equations of mathematical physics (deterministic and random), for example:

- Navier-Stokes dynamics with a view on turbulence.
- Non-linear transport and diffusion equations.
- KdV-equation and the fourth-order Kuramoto-Sivashinsky equations.
- Classical N-body dynamical equations.
- Schrödinger-type equations (linear and non-linear), including scalar-relativistic versions.

These techniques (exhibiting the far range of the decomposition method), so it appears, are presently in a testing stage as far as computer programming is concerned: the practicability of Adomian's method (considering computation time, etc.) should then be thoroughly compared with alternative methods currently in use, such as multigrid-algorithms and finite element methods, etc., most of which are computationally rather intensive due to discretization. By contrast Adomian sets out for continuous solution-processes with generally rapid convergence (here further studies are needed within the framework of functional analysis), and one may thus hope for less laborious computations in many areas of pure and applied mathematics.

I recommend Adomian's new book to all researchers in the area of mathematical modelling and solving complex dynamical systems.

There are now computational results and highly favourable comparisons of the Adomian decomposition method in numerical applications.

In recent years, he turned his attention to such fundamental problems as the origins of chaos, solution of the N-body problem, solution of Navier-Stokes and compressible Navier-Stokes equations, and the reformulation of quantum theory.